

The Harmonic Series Through the 12th Partial

Calculating from "A4" = 440, $6/5 \times 440 = \text{"C5"} = 528$, or $3/5 \times 440 = 264 = \text{"C4"}.$
Equal-tempered frequencies appear in parentheses below.

X 2X 3X 4X 5X 6X 7X 8X 9X 10X 11X 12X

| | | | | | | | | | | | | |
|--------------|----------|----------|-------|----------|----------|-------|----------|----------|----------|----------|----------|----------|
| Frequencies: | 66 | 132 | 198 | 264 | 330 | 396 | 462 | 528 | 594 | 660 | 726 | 792 |
| | (65.406) | (130.81) | (196) | (261.63) | (329.63) | (392) | (466.16) | (523.55) | (587.33) | (659.26) | (739.99) | (783.99) |

The "Major Scale" can be derived from the 4th 5th and 6th partials of three harmonic series: the one on "X," in this case "C," the one on "3X/2" ("G") and "2X/3" ("F"), resulting in "Just Intonation."

Mathematics of the Frequency and Intervallic Ratios in Just Intonation:

| | | | | | | | | |
|------------------|-----|------|-------|-----|------|-----|-------|----|
| pitch names: | do | re | mi | fa | sol | la | si | do |
| frequencies: | 1 | 9/8 | 5/4 | 4/3 | 3/2 | 5/3 | 15/8 | 2 |
| interval ratios: | 9/8 | 10/9 | 16/15 | 9/8 | 10/9 | 9/8 | 16/15 | |
| if "do" = 24 : | 24 | 27 | 30 | 32 | 36 | 40 | 45 | 48 |

Arithmetic Mean: $m - a = a - n$
 Harmonic Mean: $1/m - 1/h = 1/h - 1/n$
 therefore
 $a = 1/2(m+n)$
 $h = 2mn/m+n$

Both types of mean applied to the interval of an octave:
 $a = (1+2) / 2 = 3/2$ (perfect fifth)
 $h = 2 \times 1 \times 2 / 1 + 2 = 4/3$ (perfect fourth)

Both types of mean applied to the interval of a fifth:
 $a = (1 + 3/2) / 2 = 5/4$ (major third)
 $h = 2 \times 1 \times 3/2 / 1 + 3/2 = 6/5$ (minor third)

Major Triad is $1 : 5/4 : 3/2 = 40:50:60$
 Minor Triad is $1 : 6/5 : 3/2 = 40:48:60$